

Relationships between student throughput variables and properties

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Many different models have been designed to describe the plethora of factors that influence student throughput and success and how these factors affect throughput system variables and properties. System variables include headcounts (H) and successful credits (S) of throughput systems; some examples of system properties are the percentage of the new student intake graduating annually, and the average number of years to graduate or to drop out of a degree. However, no past study has defined the analytical relationships between these variables and properties from a process perspective – which was the purpose of this study. Three simple analytical equations were derived for 4-year degrees, and then geometrically interpreted. The behaviour of a simplified throughput system can be described by the position of a point in the admissible region of the H-S plane, with each point relating to a specific set of system properties. The successful credits ratio (S/H) is shown to be the ideal process efficiency ratio for throughput systems. The results were also extended to degrees of shorter duration. The behaviour of real throughput systems is broadly found to be similar to the behaviour of simplified throughput systems. In this study, only the mathematical foundations for the general relationships between throughput properties and throughput variables for a degree were established. The way in which this mathematical basis finds application in practice is illustrated for a few selected cases only, because of the specific focus of this paper.

Introduction

Understanding the complexities of the throughput patterns of students enrolled for degree studies at universities has always been a major challenge. Numerous models are reported in the literature to describe the plethora of factors that influence student throughput and success and how these factors affect throughput system variables and properties. A review of the current literature in this regard can be found in recent studies.^{1,2} From a process perspective, however, no study could be found that defines the analytical relationships between the most important variables and properties of a student throughput system; this absence is the reason for the current study. The throughput variables included in this study were the headcount number of students, the total number of successful module credits earned by students and the full-time equivalent (FTE) value for students enrolled for the degree. The annual intake of new students was also a special throughput variable considered here. Throughput properties considered were the percentage of new entrants graduating annually as well as the average time to graduate or to drop out of a degree. The word 'degree' in this paper is used in a generic sense and also refers to a diploma or certificate. In this paper, a simplified cohort survival model for a 4-year degree will be used to establish analytical formulae for the key system variables as functions of the system properties and the annual intake of new students. A geometrical interpretation of the analytical equations will also be given to show exactly how all the system variables and system properties are connected to one another simultaneously. The results developed for a 4-year degree will then be extended to degrees of shorter duration. This paper only establishes the mathematical basis for the general relationships between throughput properties and throughput variables for a degree. Although the application of the results to real throughput systems will be discussed, the way in which this mathematical basis finds application in practice will be illustrated for a few selected cases only. The theory as presented is not a model of South African higher education, but can be used to assess throughput process efficiency within the local higher education system as well as in other systems. The many interventions that can be undertaken by universities to improve the efficiency of the throughput process are equally important but are excluded from the scope of this paper.

Simplified cohort survival model calculations

The background calculations in this paper are structured in terms of a standard cohort survival model for a 4-year degree with a maximum time for completion assumed to be 8 years. The 8 years cut-off is necessary to ensure that the analytical expressions that will be derived are of a simple form. Treating the number of cut-off years as a variable increases the number of parameters in the model without necessarily contributing to a better understanding of throughput systems. In a simplified student throughput system, the cohort size remains constant, and the student graduation and dropout patterns for cohorts repeat year after year. These characteristics give rise to stationary throughput patterns that have also been referred to as equilibrium throughput patterns.³ The standard cohort survival model used, together with the simplifying assumptions made in this paper, created the simplest model possible for deducing analytical expressions for the various throughput variables in terms of throughput properties. Cohort survival models in general produce numbers as outcomes with very little chance of discovering the analytical relationships that exist between the said entities. The details of the cohort calculations documented in this paragraph as new research results are important in order to understand the full impact of this work, but may be skipped by readers not necessarily interested in such detail. A summary is provided in the next paragraph.

In the simplified cohort survival model, the student throughput history for a 4-year degree will be reflected by the simultaneous presence of a set of eight cohorts with cohort 1 being the youngest cohort in the year under observation. Each cohort has the same throughput profile characterised by the percentage of the cohort graduating or dropping out from the system at the end of the year. Hence, if G_i is the percentage of the annual intake N of

new students graduating and D_i the percentage of the annual intake of new students dropping out from cohort i at the end of the year, then the following equations will apply:

$$(G_4 + \dots + G_8)N = GN \text{ and } (D_1 + \dots + D_8)N = (1-G)N, \quad \text{Equation 1}$$

where GN is the number of graduates and G is the percentage of the annual intake of new students graduating at the end of the year. The very restrictive nature of Equation 1 already reflects the assumptions underlying the simplified student throughput model. More general expressions can be provided but will only increase the complexity of the analytical expressions to be derived without necessarily contributing to a better understanding of student throughput systems. Furthermore, the average number of years J taken by students to graduate at the end of the year is given by:

$$J = (4G_4 + 5G_5 + 6G_6 + 7G_7 + 8G_8)/G, \quad \text{Equation 2}$$

and the average number of years K studied by students dropping out at the end of the year is derived similarly as:

$$K = (1D_1 + 2D_2 + \dots + 7D_7 + 8D_8)/(1-G). \quad \text{Equation 3}$$

The headcount H_6 of students in the throughput system who will eventually graduate can be derived through the cohort approach as:

$$H_6 = (4G_4 + 5G_5 + 6G_6 + 7G_7 + 8G_8)N = GJN. \quad \text{Equation 4}$$

A similar expression for the headcount H_D of students who will eventually drop out of the throughput system can be derived through the cohort approach as:

$$H_D = (1D_1 + 2D_2 + \dots + 7D_7 + 8D_8)N = (1-G)KN. \quad \text{Equation 5}$$

The total headcount H for the throughput system is therefore defined in terms of the three independent system properties G , J and K as:

$$H = [GJ + (1-G)K]N = [K + G(J-K)]N. \quad \text{Equation 6}$$

This formula resembles a similar formula that was derived in a different way by Breneman for a production function for PhDs, as reported in Hopkins⁴.

To calculate the total successful module credits earned annually by students who will eventually graduate (a maximum of 1 credit per student per year), the following is considered: a student who graduates in 4 years would earn 4/4 successful module credits per year; a student who graduates in 5 years would earn an average of 4/5 successful module credits per year; etc. The total number of successful module credits S_6 earned annually by students who will eventually graduate is then given by:

$$S_6 = (4/4)(4N)G_4 + (4/5)(5N)G_5 + \dots + (4/8)(8N)G_8 = 4GN. \quad \text{Equation 7}$$

In the case of students eventually dropping out of the throughput system, the successful module credits S_D in total earned annually by these students is:

$$S_D = C(1-G)KN = 0.25(1-G)KN, \quad \text{Equation 8}$$

where C is the average number of successful credits earned by each of these students per year. For simplicity, C is assumed to be equal to 0.25 to ensure that in the case of $K=8$, in which all students that eventually drop out stay on for 8 years, the students will each have accumulated only 2 successful credits on average. This average is presumably the result of some dropouts earning up to 4 successful credits and others

earning very few credits. More realistic assumptions about the functional form of C have been tried, but these assumptions only increased the analytical complexity of Equation 8 without changing the broad conclusions reached in this paper. The total successful module credits (S) earned annually by all students is now defined in terms of the two independent system properties G and K , as:

$$S = [4G + 0.25(1-G)K]N. \quad \text{Equation 9}$$

The relationship between S and G is a linear relationship which shows a relatively direct response between S and G , especially for low values of K .

Throughput system variables and properties in summary

The number of graduates produced by the simplified throughput system at the end of the year under observation would be equal to GN – where N is the annual intake of new students and G is the percentage of the annual intake of new students graduating (ranging between 0% and 100%). The average number of years taken by students who have graduated at the end of the year under observation is denoted by J . Similarly, the average number of years studied by students dropping out at the end of the year under observation is denoted by K . The value of J ranges between 4 and 8, and the value of K ranges between 1 and 8. The three quantities G , J and K are independent of one another. They also describe the main characteristics of the student throughput process and are therefore referred to as throughput system properties. The characteristics of a throughput system can therefore be described in terms of all the possible combinations of G , J and K . System variables are used to describe bulk system quantities that only change if the properties of the throughput system change. These system variables are the headcount H of the throughput system, the successful module credits S annually earned by students, and the FTE value for students enrolled for the degree V . It has already been shown in Equations 6 and 9 that the system variables H and S depend on the system properties G , J and K and on the annual intake of new students N . Student headcount H , which in the South African higher education system refers to an unduplicated count of students irrespective of the academic course load of the student, depends on G , J and K as follows:

$$H = [GJ + (1-G)K]N = [K + G(J-K)]N. \quad \text{Equation 6}$$

The perfect throughput system in which all students graduate in minimum time, has a size equal to $H=4N$ when $G=100\%$ and $J=4$. The influence of K diminishes as G approaches 100%. The total successful module credits S earned annually by students enrolled for a 4-year degree (a maximum credit of 1 per student per year) depends on G and K as follows:

$$S = [4G + 0.25(1-G)K]N. \quad \text{Equation 9}$$

The number of successful module credits S earned annually by students for the perfect throughput system is equal to $S=4N$ when $G=100\%$. Again, the influence of K diminishes as G approaches 100%. Apart from the successful credits (S) earned annually by students, the credits assigned to modules not successfully passed by students in the same year will be referred to here as failed credits (F). Furthermore, some students often take fewer modules than required by a full academic load with module credits therefore adding up to less than 1. In this paper, the balance of module credits not attempted by students in a particular year is referred to as unutilised credits (U). Using the fact that each of the H headcount students can at most generate 1 credit per year, it clearly follows that:

$$S + F + U = H \text{ or } F + U = H - S. \quad \text{Equation 10}$$

The FTE value for students enrolled for the degree V as a throughput variable, which in the South African higher education system depends on

for completion of a degree in general, J will then range between M and $2M$, whereas K would range between 1 and $2M$. The headcount H of the student throughput system is given by $H = [K + G(J - K)]N$, whereas the successful module credits S earned annually by students would then be given by $S = [MG + 0.25(1 - G)K]N$.

Application of the findings to a few selected topics

The main purpose of this paper is to establish the mathematical basis for the general relationships between throughput properties and throughput variables for a degree. However, a few simple applications of the equations derived in this paper will now be given for degrees offered in the South African higher education system. More complex applications to issues such as the conversion of 3-year degrees into 4-year degrees for students who are expected to find it difficult to complete the 3-year degree in minimum time,⁵ will be discussed in a follow-up paper.

Clarity on process efficiency measurements

In the South African higher education system, the success rate $S/(S + F)$ that can be written as $S/(S + F) = (S/H)/[1 - L(1 - S/H)]$ is used as a measure of efficiency instead of the successful credits ratio (S/H) as defined in this paper. The success rate will clearly measure the process efficiency of degrees differently because of its dependence on L . Degrees with L values close to 1 and a success rate therefore approximately equal to 1 , will seem to be more efficient than degrees with L values close to 0 and a success rate therefore equal to approximately S/H . This difference in efficiency measurement becomes very pronounced in the case of degrees with relatively low successful credits ratios, such as $S/H = 50\%$. The use of the success rate should therefore be discontinued in favour of the use of the successful credits ratio, S/H .

Another ratio that is often used in the South African higher education system as a measure for process efficiency is the FTE to headcount ratio (V/H) . With V equal to $LS + (1 - L)H$, it is clear that this ratio would likewise produce different measurements of process efficiency as a result of its dependence on L ; its use should therefore also be discontinued.

The graduate output to size ratio (GN/H) is known in the South African higher education system as the graduation rate, and its application to real throughput systems is widely contested because of fluctuations in both the values of G and the intake N of new students.⁶ Despite this disadvantage in the case of real systems, it may still be useful in simplified systems as an absolute output to size ratio, focusing on the process efficiency of the teaching learning process irrespective of the duration of the degree. However, a 4-year degree with GN/H being equal to 0.25 at best, would then from a process perspective appear to be less efficient than a 3-year degree with its ratio GN/H being equal to 0.33 at best. This outcome would certainly restrict the use of this ratio as a process efficiency measurement for degrees in general. The general perception that GN/H is an unrelated or even a more comprehensive measurement of process efficiency is also incorrect. The fact is that the two measurements (S/H) and (GN/H) are related to one another through Equation 9 with $(GN/H) = (S/H)/4 = 0.25$ for the perfect throughput system. It is noted that lines of $GN/H = \text{constant}$ all pass through the fictitious origin of the H - G plane; in particular, the line $GN/H = 0.133$ passes through Y in Figure 1.

Inequities in the funding framework for South African universities

The biggest part of the funding framework for South African universities is based on the sum total of the FTE values $V = LS + (1 - L)H$ for each university degree. Degrees with L values close to 1 will have FTE values very close to S and therefore a funding base that will be largely performance driven through the output variable S . However, degrees with L values close to 0 will have FTE values very close to H and therefore a funding base that will be largely input driven through the input variable H . This difference is considerable, especially in the case of degrees in which the successful credits ratio is $S/H = 50\%$. In such a case, the $L = 0$

degree will receive double the funding received by the $L = 1$ degree. The question is whether two degrees with exactly the same successful credits ratio S/H should be funded at such disparate levels especially if no specific reason can be identified to justify the existence of different values of L ? It therefore seems reasonable that L should be assumed to be 0.5 for all degrees and that the quantity $LS + (1 - L)H = (S + H)/2$ be used as a more appropriate basis for the funding of degrees offered by South African universities. This approach would result in all degrees being funded on the basis of their average of S and H , which, however, does not affect the actual L value for the degree, although one may eventually find a tendency amongst faculties to manage the L values of their degrees towards $L = 0.5$. Consideration could perhaps also be given to change the definition of the FTE value to be based on the average of S and H . It is noted that, whereas H can be regarded as the nominal size of the degree relating to an unduplicated count of student names on a list, the quantity S can be regarded as the credit earning size of the degree. The redefined FTE values $(S + H)/2$ could then be regarded as the effective size of the degree. Again, determining the full implications for the funding of South African higher education should, because of the complexity of the topic, require much more research and should rather be pursued outside of the scope of this introductory paper.

Enrolment management

The size of the South African higher education system is currently regulated by government-approved enrolment plans for universities with quotas for both H and V set for each university for each year. A minor relaxation of these constraints has recently been proposed,⁷ which will not address the difficulties highlighted below. The enrolment plans also call for meaningful annual improvement of the success rates and graduation rates for each university, thus signalling that higher successful credits ratios (S/H) need to be achieved for each university. These enrolment plans have three unintended consequences. Firstly, by imposing the FTE values constraint on throughput systems, universities would have to manage the H part of this constraint at the beginning of each academic year according to Equation 11 but would only know the value of the S part at the end of the year as a consequence of successful teaching outcomes. Such a constraint is very difficult to manage in practice. Secondly, when written in the form $V/H = [1 - L(1 - S/H)]$, Equation 11 states that a student throughput system in which both V and H are constant, or even directly proportional to one another, can only produce the same but not higher successful credits ratios (S/H) as required by the enrolment plans. This dilemma can apparently be resolved by only retaining the constraint on V , which is the more important constraint relating to the funding of the system. However, this again leaves the university with a constraint which is very difficult to manage in practice. Thirdly, imposing headcount quotas on the throughput system has led to enrolment management practices at South African universities which amount to registering returning students first and then using the new student intake (N) to make up for the shortfall. This unfortunate way of setting the size of the new student intake (N) introduces awkward fluctuations into the student throughput patterns during subsequent years, which in turn undermines proper planning with regard to the provision of facilities and lecturing staff. These three unintended consequences of enrolment planning within the South African higher education system, also point to the need for further research to be undertaken to resolve these difficulties.

Real student throughput systems

A better understanding of real throughput systems should follow directly from a study of simplified systems. Simplified systems are not theoretical constructs but in fact special cases of real throughput systems. The behaviour of a simplified throughput system can be described by the position of a point in the admissible region of the H - S plane, with each point relating to a specific set of system properties. The successful credits ratio of the throughput system is equal to S/H with the redefined FTE values equal to $(S + H)/2$. The same behaviour, however, is observed from the data for real throughput systems. This is explained by the fact that H can be calculated independently of Equation 6 by simply adding together the actual unduplicated number of students enrolled for a

degree. The same calculation can be performed for S independent of Equation 9. Hence, for all real throughput systems the successful credits ratio would be defined by S/H with the redefined FTE value equal to $(S+H)/2$. Only in the case of a constant annual intake of new students N , would it be possible to define the position of the perfect throughput system $H=4N=S$. It is also noted that these calculations may now be performed at the university level because both H and S can be added together for a group of degrees.

Conclusions

In this paper, the mathematical foundation for the general relationships between throughput properties and throughput variables for a degree has been established using a simplified or equilibrium cohort survival approach. The simplified model assumes a constant annual intake of new students and that throughput system properties such as the graduation and dropout patterns for each cohort also remain the same. Throughput properties include the percentage G of the annual intake of new students graduating annually as well as the average number of years J to graduate and the average number of years K to drop out of a degree. The balance L between the unutilised and failed module credits within the throughput system is the fourth system property required. Three analytical formulae have been derived for important system variables, such as the headcounts H and total successful module credits S of the throughput system, both of which depend on the system properties G , J and K as well as on the annual intake of new students N . The FTE value V has been expressed in terms of S , H and L .

In this paper, it has been demonstrated that the system variables H and S are not independent but are each dependent on the same set of G , J and K values for the simplified student throughput configuration under consideration. Furthermore, only certain H and S values can simultaneously be realised – namely those in the admissible region of the H - S plane. The admissible region has a triangular shape bounded by edges corresponding to the maximum and minimum values of J and G . The shape of the triangle is determined by the value of K , which should preferably be managed to be equal to the minimum time for completion of the degree. Furthermore, within this triangle, the relationships between throughput variables and throughput properties become visible through suitable geometrical constructions. In essence, the behaviour of a simplified throughput system can be described by the position of a

point in the admissible region of the H - S plane, with each point relating to a specific set of system properties. The migration of the throughput system in the H - S plane means changing the system properties and allowing enough time for the system to establish equilibrium in its throughput patterns. The successful credits ratio S/H produces a simultaneous account of the combined efficiency status of the three system properties G , J and K . More importantly, the FTE number of students is given by $(S+H)/2$ for $L=0.5$, and for a given value of N , the perfect throughput system for a 4-year degree is located at $H=4N=S$ with $S/N=100\%$. This paper provides indications on how the simplified model for a 4-year student throughput system can be changed to apply to degrees of different duration.

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